

Calcul des déformations des ressorts hélicoïdaux

Ressort cylindrique - Force et couple concentrés

Flexion et torsion - Exemple numérique

Lame en acier de section rectangulaire, h dans le plan vertical, $h > e$

$$h := 10 \cdot \text{mm} \quad e := 5 \cdot \text{mm} \quad S := h \cdot e \quad E := 2 \cdot 10^5 \cdot \text{N} \cdot \text{mm}^{-2} \quad G := \frac{E}{2.6} \quad \rho := 7.85 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

➔ Référence : E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$J_t := J_{t_rect}(h, e) \quad I_{22} := I_{f_rect}(h, e) \quad I_{33} := I_{f_rect}(e, h)$$

$$W_t := W_{t_rect}(h, e) \quad W'_t := \frac{W_t}{\eta_{t_rect}(h, e)} \quad W_{f2} := W_{f_rect}(h, e) \quad W_{f3} := W_{f_rect}(e, h)$$

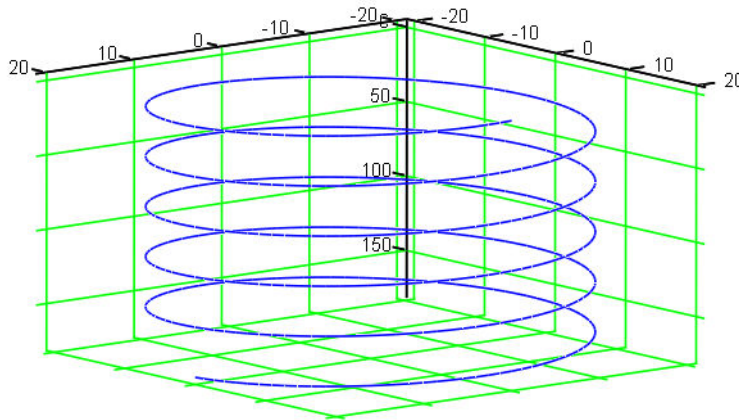
Caractéristiques du ressort $D := 40 \cdot \text{mm} \quad R := 0.5 \cdot D \quad n_{sp} := 5.25 \quad \psi_{AB} := 2 \cdot \pi \cdot n_{sp}$

$$\beta := 15 \cdot \text{deg} \quad p := 2 \cdot \pi \cdot R \cdot \tan(\beta) \quad s(\alpha) := R \cdot \alpha \cdot \cos(\beta)^{-1} \quad L := s(\psi_{AB}) \quad L = 68.301 \text{ cm}$$

Forme du ressort

$$x_0(\alpha) := R \cdot \cos(\alpha) \quad y_0(\alpha) := R \cdot \sin(\alpha) \quad z_0(\alpha) := R \cdot \tan(\beta) \cdot \alpha$$

$$n := 100 \cdot n_{sp} + 1 \quad i := 0 \dots n - 1 \quad \alpha_{0_i} := \frac{\psi_{AB}}{n - 1} \cdot i \quad x_i := x_0(\alpha_{0_i}) \quad y_i := y_0(\alpha_{0_i}) \quad z_i := z_0(\alpha_{0_i})$$



$$\left(\frac{x}{\text{mm}}, \frac{y}{\text{mm}}, \frac{z}{\text{mm}} \right)$$

Forces extérieures excentrées à l'extrémité inférieure du ressort $\psi_F := \psi_{AB} \quad \chi_F := \arctan\left(\frac{R}{2 \cdot |z_0(\psi_F)|}\right)$

$$P := 50 \cdot \text{N} \quad r_F := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_F := 60 \cdot \text{deg} \quad \gamma_F := 45 \cdot \text{deg} \quad \chi_F = 3.238 \text{ deg}$$

$$\mathbf{F} := P \cdot \begin{pmatrix} \cos(\lambda_F) \cdot \sin(\gamma_F) & \sin(\lambda_F) \cdot \sin(\gamma_F) & \cos(\gamma_F) \end{pmatrix}^T \quad \mathbf{F}^T = (17.678 \quad 30.619 \quad 35.355) \text{ N}$$

$$C := 5 \cdot \text{N} \cdot \text{m} \quad r_C := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_C := 150 \cdot \text{deg} \quad \gamma_C := 45 \cdot \text{deg}$$

$$\mathbf{C} := C \cdot \begin{pmatrix} \cos(\lambda_C) \cdot \sin(\gamma_C) & \sin(\lambda_C) \cdot \sin(\gamma_C) & \cos(\gamma_C) \end{pmatrix}^T \quad \mathbf{C}^T = (-3.062 \quad 1.768 \quad 3.536) \text{ m N}$$

➡ Référence :E:\Résonateur (TA)\Ressorts hélicoïdaux\Ressort hélicoïdal E_L - F&C.mcd(R)

Torseur des forces de cohésion

$$\alpha_M := 40 \cdot \text{deg} \quad \mathbf{R}_c(\alpha_M)^T = (17.678 \quad 30.619 \quad 35.355) \text{ N}$$

$$\mathbf{M}_c(\alpha_M)^T = (-8.461 \quad 5.368 \quad 3.117) \text{ N} \cdot \text{m}$$

Sollicitations

Traction-compression $N_c(\alpha_M) = 20.831 \text{ N}$

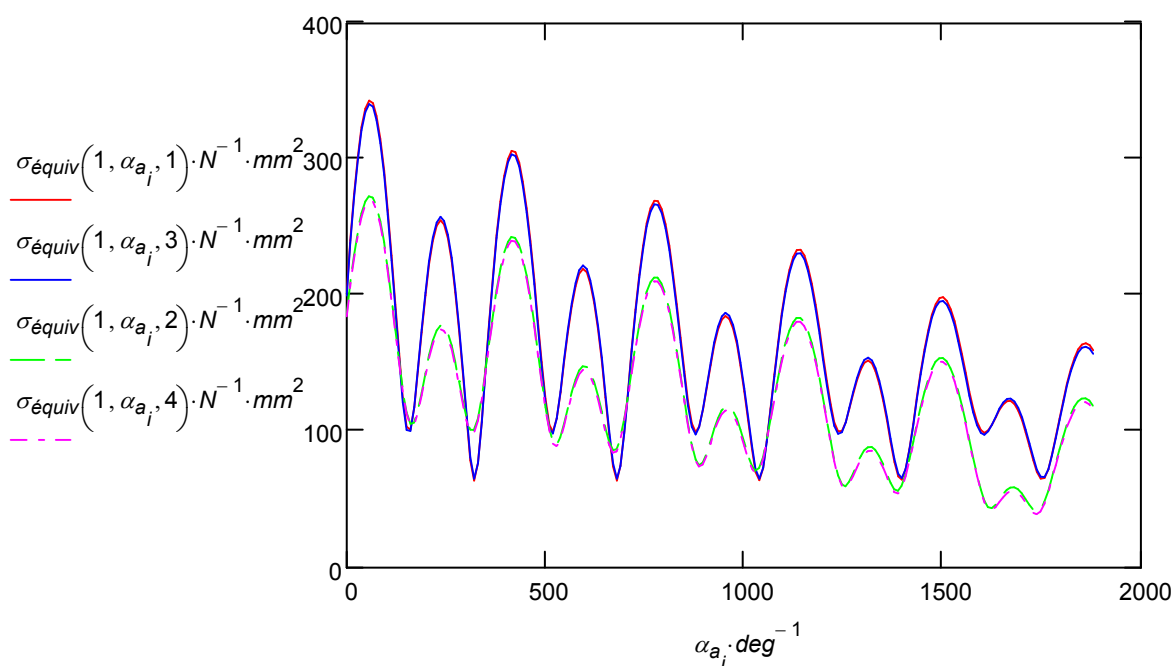
Efforts tranchants $Q_2(\alpha_M) = -33.223 \text{ N} \quad Q_3(\alpha_M) = 31.021 \text{ N}$

Moment de torsion $M_t(\alpha_M) = 10.032 \text{ N} \cdot \text{m}$

Moments de flexion $M_{f2}(\alpha_M) = 3.031 \text{ N} \cdot \text{m} \quad M_{f3}(\alpha_M) = 0.539 \text{ N} \cdot \text{m}$

Contraintes

$$n := 201 \quad i := 1 \dots n - 1 \quad \alpha_{a_i} := (i - 1) \cdot \frac{\psi_{AB}}{n - 1}$$



Calcul des déplacements linéiques

Position du déplacement désiré

$$\alpha_M := \psi_{AB}$$

Déplacement dans la direction de Ox

$$\lambda := 0 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 5.194 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 1.304 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = -1.564 \text{ mm}$$

$$\delta_x(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_x(\alpha_M) = 4.934 \text{ mm}$$

Déplacement dans la direction de Oy

$$\lambda := 90 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 8.349 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 2.237 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = 0.782 \text{ mm}$$

$$\delta_y(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_y(\alpha_M) = 11.369 \text{ mm}$$

Déplacement dans la direction de Oz $\lambda := 0 \cdot \text{deg}$ $\gamma := 0 \cdot \text{deg}$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = -0.548 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = -0.436 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = -0.583 \text{ mm}$$

$$\delta_z(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_z(\alpha_M) = -1.567 \text{ mm}$$

Calcul des déplacements angulaires

Déplacement angulaire autour de Ox $\lambda_c := 0 \cdot \text{deg}$ $\gamma_c := 90 \cdot \text{deg}$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = -4.726 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = -1.248 \text{ deg}$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = -0.223 \text{ deg}$$

$$\theta_x(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_x(\alpha_M) = -6.197 \text{ deg}$$

Déplacement angulaire autour de Oy $\lambda_c := 90 \cdot \text{deg}$ $\gamma_c := 90 \cdot \text{deg}$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 2.845 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0.743 \text{ deg}$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = 0.316 \text{ deg}$$

$$\theta_y(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_y(\alpha_M) = 3.905 \text{ deg}$$

Déplacement angulaire autour de Oz $\lambda_c := 0 \cdot \text{deg}$ $\gamma_c := 0 \cdot \text{deg}$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 0.845 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0 \text{ deg}$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = 5.401 \text{ deg}$$

$$\theta_z(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_z(\alpha_M) = 6.246 \text{ deg}$$

Graphe de la déformation

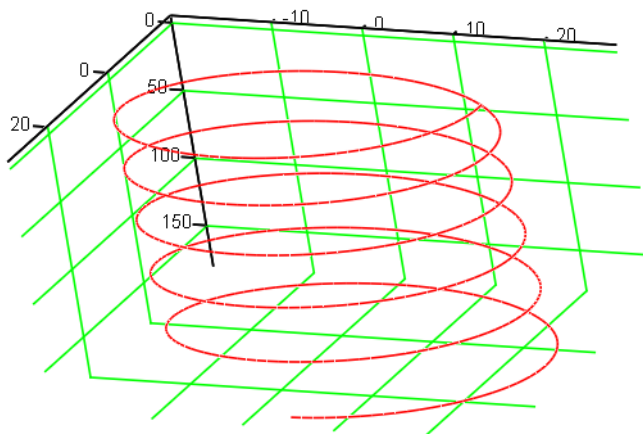
$$x_d(\alpha) := x_0(\alpha) + \delta_x(\alpha)$$

$$y_d(\alpha) := y_0(\alpha) + \delta_y(\alpha)$$

$$z_d(\alpha) := z_0(\alpha) + \delta_z(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \quad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$$X := \overrightarrow{x_d(\alpha_0)} \quad Y := \overrightarrow{y_d(\alpha_0)} \quad Z := \overrightarrow{z_d(\alpha_0)}$$



$$\left(\frac{X}{\text{mm}}, \frac{Y}{\text{mm}}, \frac{Z}{\text{mm}} \right)$$